

**REPORT DOCUMENTATION PAGE**Form Approved  
OMB No. 074-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing this collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503

<b>1. AGENCY USE ONLY (Leave blank)</b>		<b>2. REPORT DATE</b> 1994	<b>3. REPORT TYPE AND DATES COVERED</b> Conference proceeding, 4-6 October 1994	
<b>4. TITLE AND SUBTITLE</b> Computer Aided Design For Reactive Power Compensation			<b>5. FUNDING NUMBERS</b> N/A	
<b>6. AUTHOR(S)</b> N.K. Swain, N.N. Bengiamin and F.H. Holcomb				
<b>7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)</b> Electrical Engineering Department, University of North Dakota Grand Forks, ND 58202 Energy & Utility Systems Division U.S. Army, Construction Engineering Research Laboratories Champaign, IL 61821			<b>8. PERFORMING ORGANIZATION REPORT NUMBER</b> N/A	
<b>9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)</b> SERDP 901 North Stuart St. Suite 303 Arlington, VA 22203			<b>10. SPONSORING / MONITORING AGENCY REPORT NUMBER</b> N/A	
<b>11. SUPPLEMENTARY NOTES</b> Presented at the 30th Annual Minnesota Power Systems Conference Proceedings, Earle Brown Continuing Education Center, St. Paul Campus, University of Minnesota, 4-6 October 1994. This work was supported in part by SERDP. The United States Government has a royalty-free license throughout the world in all copyrightable material contained herein. All other rights are reserved by the copyright owner.				
<b>12a. DISTRIBUTION / AVAILABILITY STATEMENT</b> Approved for public release: distribution is unlimited			<b>12b. DISTRIBUTION CODE</b> A	
<b>13. ABSTRACT (Maximum 200 Words)</b>  Presented in this paper is a computer aided design tool to determine the size and location of shunt capacitors for the purpose of reducing power losses in distribution networks. The developed computer program (CapSize) is windows based with several design options that permit practically feasible solutions. The implemented algorithm is based on a rigorous optimization method and ac power flow calculations. The algorithm accounts for voltage profile, load variations, load transfer switching, observation measurements, and limited availability of data. The results of a test case are presented for illustration.				
<b>14. SUBJECT TERMS</b> CapSize, reactive power compensation, SERDP			<b>15. NUMBER OF PAGES</b> 11	
			<b>16. PRICE CODE</b> N/A	
<b>17. SECURITY CLASSIFICATION OF REPORT</b> unclass	<b>18. SECURITY CLASSIFICATION OF THIS PAGE</b> unclass	<b>19. SECURITY CLASSIFICATION OF ABSTRACT</b> unclass	<b>20. LIMITATION OF ABSTRACT</b> UL	

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)  
Prescribed by ANSI Std. Z39-18  
298-102

DTIC QUALITY INSPECTED 1



# Minnesota Power Systems Conference Proceedings

**October 4-6, 1994**

**Earle Brown Continuing Education Center,  
St. Paul Campus, University of Minnesota**

Co-sponsored by:

Twin Cities Section of IEEE

and Professional Development and Conference Services,

Continuing Education and Extension,

University of Minnesota

UNIVERSITY OF MINNESOTA

# *Computer Aided Design For Reactive Power Compensation*

N.K. Swain<sup>1</sup>   N. N. Bengiamin<sup>1</sup>   F.H.Holcomb<sup>2</sup>

<sup>1</sup>Electrical Engineering Department  
University of North Dakota  
Grand Forks, ND 58202 USA

<sup>2</sup>Energy & Utility Systems Division  
U.S. Army  
Construction Engineering Research Laboratories  
Champaign, IL 61821 USA

# *Computer Aided Design For Reactive Power Compensation*

N. Swain, N.N. Bengiamin and F.H. Holcomb

## **Abstract**

Presented in this paper is a computer aided design tool to determine the size and location of shunt capacitors for the purpose of reducing power losses in distribution networks. The developed computer program (CapSize) is windows based with several design options that permit practically feasible solutions. The implemented algorithm is based on a rigorous optimization method and ac power flow calculations. The algorithm accounts for voltage profile, load variations, load transfer switching, observation measurements, and limited availability of data. The results of a test case are presented for illustration.

## **1. Introduction**

The usual method of ohmic loss reduction in distribution networks is through the use of shunt capacitors. The shunt capacitors reduce the reactive component of power flow, and hence lead to a better power factor and voltage profile. Ohmic losses are due to the flow of real and reactive power in conductors. The former can not be reduced since real power is the useful work producing component and it has to flow the length of the feeder from the energy source to the load. The losses due to reactive power flow can theoretically be reduced to zero by placing capacitors at each node with the size of capacitor being equal to the reactive component of load at that node. Although this method provides the minimum reactive system losses, it is not practical because the cost of compensating capacitors may exceed the benefits gained from reducing these losses. This may also lead to excessive and non economical capacitor switching. Therefore, a compromise solution, that will address these issues and account for load variations, is needed.

Several methods of loss reduction in distribution systems are reported in the literature [1, 2]. These techniques vary in complexity according to the assumptions made and system details accounted for. Assuming a uniform conductor with uniformly distributed loads and no laterals, a rule of thumb (the famous 2/3 rd rule) suitable for hand calculations was possible. With the advent of computer technology this limiting assumption became unpractical and can hardly be used for any realistic situation. The more advanced techniques have all required solving a non-linear optimization problem with the combined demand and energy costs forming the objective function.

Heuristic methods are often suggested to reduce computational complexities. Enumerative techniques such as branch and bound or even explicit search are used. More rigorous schemes are of the iterative type. Researchers did not spare much effort formulating and solving ohmic loss reduction problems with voltage dependence [3], feeder laterals, switching capacitors [4], phase unbalance [6], voltage regulators, transfer switches automation [7], and load variations included [1]. The computation time escalates rapidly due to complexity of the employed search schemes. The authors are aware of only one commercially available computer program for capacitors location and sizing; it is of the explicit search type.

This paper presents a design tool that permits a successive evolution in design to satisfy practical constraints. The design engineer is able to alter the ideal theoretical design and tailor it according to his/her particular engineering judgment and desired outcomes.

The implemented algorithm is an extension to the fundamental work by Grainger and Lee [3, 6]. The optimization algorithm utilizes normalized equivalent feeder data to easily account for non-uniformity in feeder configuration, and ac power flow to account for voltage variations. The search technique is iterative and it progresses to satisfy an equal area criterion for current distribution as the reactive current curve shifts with the addition of new shunt capacitors.

## 2. Design Methodology

The original work by Grainger and Lee [3, 6] assumed radial feeders. This assumption is modified in this work to include laterals. Each lateral is treated as a radial feeder for which the number of capacitors, their sizes and locations, loss reduction and savings are determined. Depending on the savings and the size (a user defined parameter), either all the loads or a load equivalent (difference between the load and the capacitor) is then concentrated at the lateral feed point on the main feeder. This process is repeated for all laterals and sub-laterals until the main feeder becomes a pure radial feeder. This equivalent feeder is then analyzed for reactive power compensation. The algorithm also provides the user with the option to determine the capacitor sizes and their locations for an equivalent radial feeder where all lateral loads are treated as concentrated loads at their respective lateral starting points. This is useful when the lateral losses are very small and an approximate practical solution is desirable.

Motivated by the need to accommodate load transfer without voltage profile deterioration due to overcompensation in reduced load sections, the network is divided into zones. The zones are determined based on the location of load transfer switches. Assuming open transfer switches, each load island constitutes a zone. Zone-1 is the zone that directly connected to the source.

The reactive compensation method is applied to all zones, and is carried out in the following four phases:

**Phase-1** of the solution solves the ac power flow equations to determine node voltages, branch losses, branch currents and real and reactive power at the substation.

**Phase-2** employs the normalized feeder and optimization principles of [6], uses the substation power and reactive current calculated in phase-1 and accounts for load variation of the system to determine optimum location for the capacitors. This phase may produce a near optimal solution with regard to capacitor size.

**Phase-3** solves ac power flow equations iteratively to determine the optimum size of capacitors for maximum loss reduction.

**Phase-4** determines the savings due to power and energy loss reduction, and the load equivalent,

say QIs, (difference between total reactive component of the load and the total capacitor in the zone). Also, the total real component of the load, say Pls, the zone is determined.

For the closest zone to the substation (zone-1), consider all the loads originally in that zone and loads Pls and QIs from the other zones at their respective branch points. Use the phases described above to determine the capacitor sizes, location, reduction in losses and savings.

Since each zone is considered individually for reactive compensation, the shunt capacitors will naturally be located within the zone boundaries allowing for transfer of compensation with the load. Therefore, an acceptable reactive compensation level and voltage profile are always maintained.

### 3. CapSize

CapSize is a WINDOWS-based computer program that determines the optimal number, size and location of shunt capacitors for radial distribution feeders without and with laterals. Three basic design options are available:

**Option-1)** Determines the required number, size, and locations of capacitors yielding the most optimal solution. A sub-optimal design with one capacitor is also possible using this option.

**Option-2)** Determines the optimal locations for capacitors of pre-determined size. This option is desirable if the capacitor sizes obtained from option-1 are not in the practical range.

**Option-3)** Determines the optimum size of capacitors at pre-determined locations. This sub-optimal design is useful if option-1 produced an excessive number of capacitors or the determined locations are undesirable.

The design engineer may use these options in any order to meet the particular objectives of the system. Besides these three options CapSize has also the following features:

- graphs the current distribution of the feeder with and without laterals.
- displays and prints a brief summary and a detailed report.
- edits the data files.

The three design options share some common schemes in their algorithm. These schemes will be presented first and the individual algorithms will follow.

#### Common Schemes:

##### **a.) AC Power Flow (Subprogram-APF)**

1. It determines voltage at each node, current and losses in each branch of the feeder using power flow equations; determines the total real power loss (say  $P_{\max}$ ); calculates the maximum possible real power loss reduction,

$$P_e = P_{\max} - P_{\min} \quad (1)$$

where,  $P_{\min}$  is the ohmic loss due to the flow of real power component.

### b) Feeder Parameter Normalization (Subprogram-FPN)

1. It normalizes the feeder parameters and calculates the normalized current as follows[6]:

i. Choose the resistance  $r^*$  ohm/unit length of the first section of the feeder as the base for normalizing the uniform feeder's impedance.

ii. Calculate the equivalent length "L<sub>ui</sub>" for each section (i) using

$$L_{ui} = (L_i / r_i) / r^*, i=1,2..k, \quad (2)$$

iii. Calculate the total length of the equivalent uniform feeder "L<sub>u</sub>" using

$$L_u = \sum_{i=1}^k L_{ui} \quad (3)$$

iv. Calculate the equivalent normalized length "L<sub>ni</sub>" for each section using

$$L_{ni} = L_{ui} / L_u \quad (4)$$

v. Calculate the resistance per unit length "r<sub>no</sub>" for the normalized feeder using

$$r_{no} = \sum_{i=1}^k L_i * r_i \quad (5)$$

vi. Calculate the normalized current in each section of the feeder by dividing the actual feeder current in each section (f<sub>xni</sub>) by the base reactive current, ( $Q_0 / \sqrt{3} V_0$ )

vii. Increase the reactive component of the load in each section by the corresponding section reactive losses calculated in (a).

2. Create the data file with the normalized parameters.

3. Calculate  $\alpha$  as follows:

$$\alpha = (K_p + K_e * T * L_f) / (K_p + K_e * T) \quad (6)$$

This parameter accounts for the load factor effect, where  $K_p$ = demand charge in \$/kw/yr,  $K_e$ =energy charge in mills/kwh,  $T$ =time in hrs, and  $L_f$ =annual reactive load factor.

## Individual Algorithms

### a) Option-1: Optimum Number, Location and Size

**Step -1. Call Subprogram-APF** to determine the voltage at each node, current and losses in each branch of the feeder.

**Step-2. Call Subprogram-FPN** to create the normalized parameter data file.

**Step - 3.** The objective of the capacitor placement is to maximize the net dollar savings resulting from both power and energy loss reductions and is given by,

$$S = K_p LP + K_e LE - K_c \sum_{i=1}^n Q_{ci} \quad (7)$$

where LP=total real power loss reduction, LE= total energy loss reduction, and  $Q_{ci}$ = capacitor size,  $n$  is the number of capacitors, and  $K_c$  = cost of capacitor in \$/ kvar. Assume  $K_c = 0$  and invoke the Equal Area Criterion [6] of step-4 to determine the capacitor locations and sizes to minimize power and energy losses and maximize  $S$ . This is the **near optimal solution**.

### Step - 4: Equal Area Criterion:

Determine the optimum locations " $h_i$ " and size " $Q_{ci}$ " where  $h_i$  is the distance of capacitor  $j$  from the source; Capacitor number "1" is the farthest from the source. This is done as follows:

**4a.** Select an arbitrary value for  $h_1$ ; e.g. the end of the feeder. This value determines the farthest permissible location from the source where capacitors can be located. Apply the following steps for  $i=1,2,\dots,n$ .

**4b.** Calculate  $I_{ci}$ , the current due to capacitor " $i$ " using

$$I_{ci} = 2 \left\{ \alpha I_s F(h_i) - \sum_{j=1}^{i-1} I_{cj} \right\} \quad (8)$$

where  $F(h_i)$  is the normalized reactive current at  $h_i$ . The "2" multiplier is relevant to the popular 2/3 rule in reactive power compensation. The capacitor size  $Q_{ci}$  is calculated using  $\sqrt{3} V h_i I_{ci}$ .

**4c.** Find " $g_i$ " such that

$$F(g_i) = \left( \sum_{j=1}^i I_{cj} \right) / (\alpha I_s) \quad (9)$$

where  $I_s$  is the peak reactive current injected to the feeder at the source, and  $F(g_i)$  is the normalized reactive current at location " $g_i$ " on the equivalent feeder.

**4d.** Shift the normalized feeder reactive current curve down such that  $F(g_i)=0$ , then find  $h_{i+1}$  which produces an **area "B"** under the curve equal to **area "A"** above the curve that is bounded



by  $h_i$  and  $g_i$ , Figure 1. If  $h_{i+1}$  is a viable location on the feeder, increment "i" and go to 4b.

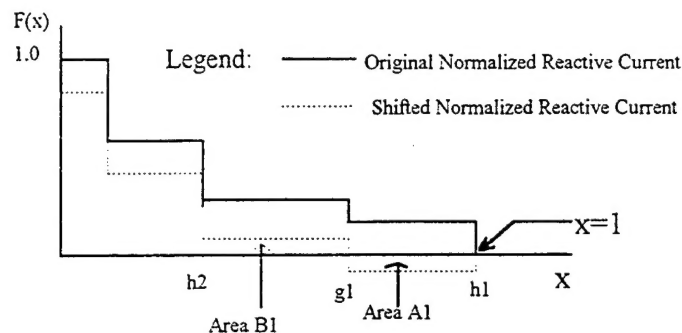


Figure 1: Illustration of Equal Area Criterion.

4e. If  $h_{n+1}=0$  and  $A_n=B_n$ , then stop, a solution has been found. Otherwise, reset "i" to 1, reduce  $h_1$  if  $B_n > A_n$  and increase  $h_1$  if  $B_n < A_n$ , then return to 4b. It must be noted that if  $h_i$  is a step feed point, the algorithm has the flexibility of adjusting  $F(h_i)$  to satisfy the equal area criterion.

**Step-5.** Convert the normalized locations  $h_i$ ,  $i=1,2,\dots,n$  to their corresponding actual locations along the feeder.

**Step - 6.** Call subprogram APF. Calculate the new losses, voltages and reactive currents. Go to step 8.

**Step - 7.** Increase the capacitor sizes by equal amount and run Subprogram APF iteratively until the total loss is less equal to  $P_e$  within tolerance. This is the **optimal solution**.

**Step - 8.** Determine the reduction in demand.

$LP = \text{loss of step-1} - \text{loss of step-7/6}.$

**Step - 9.** Determine the energy loss and reduction in energy loss.

$\text{energy loss (E)} = \text{peak power loss} \times T \times (Lf)^2$

$LE = E - \text{energy loss of step-7/6}.$

**Step - 10.** Determine the saving due to loss reduction using equation 7.

## b.) Option-2: Optimum Location

**Step-1.** Same as Step-1 of Option-1.

**Step-2.** Same as Step-2 of Option-1.

**Step-3.** Determine the location for the predetermined capacitor sizes. This is done as follows:

**3a.** Determine the normalized reactive current  $f_{hi}$  for the  $i$ th capacitor using

$$I_{ci} = (1/\alpha) * (K_{vari}/K_{vart}) \quad (10)$$

where,  $K_{vari}$  is the ratings of capacitor  $i$  and  $K_{vart}$  is the base kvar.

$$I_{cx} = \sum_{j=1}^{i-1} I_{cj}, \quad \sum_{j=1}^0 I_{cj} = 0 \quad (11)$$

$$f_{hi} = (I_{ci}/2) + I_{cx} \quad (12)$$

**3b.** Compare  $f_{hi}$  with the normalized current ( $f_{xni}$ ) of Step-2. The location for the  $i$ th capacitor is found by adding all the section lengths upto the point where  $f_{hi} = f_{xni}$ .

**Step-4.** Call Subprogram-APF and steps 8-10 to determine losses. and savings.

One of the requirement of this algorithm is that the predetermined capacitors must be of different sizes successively increasing towards the source. The locations determined by the algorithm will be always at the feed points.

### c.) Option-3: Optimum Sizes

**Step-1:** Same as Option-1.

**Step-2:** Same as Option-1.

**Step-3:** Normalize the predetermined locations for the capacitors.

**Step-4:** Calculate area  $A_{i,i+1}$  under the  $F(x)$  curve between capacitor locations  $h_i$  and  $h_{i+1}$ , using

$$A_{i,i+1} = \int_{h_{i+1}}^{h_i} F(x) dx, \text{ for } i=1, 2, \dots, n \quad (13)$$

**Step-5:** Calculate  $F(g_i)$  for the  $i$ th capacitor using

$$F(g_i) = (A_{i,i+1}) / (h_i - h_{i+1}) \quad (14)$$

**Step-6:** Calculate the size of the  $i$ th capacitor using

$$Q_{ci} = \alpha * k_{vart} * [F(g_i) - F(g_{i-1})] \quad (15)$$

and  $F(g_0) = 0$

**Step-7:** Increment  $i$ . Return to Step-4 and repeat the process until  $i=n$ , where  $n$  is the number of predetermined locations.

**Step-8:** Call Subprogram-APF and steps 8-10 to determine Losses and Savings.

#### 4. Test Case

Depicted in Figure 2 is the one line diagram of a test feeder with three distinct zones of loading. This feeder is simulated using the CapSize computer program, with a substation voltage of 12 kv, load factor of .95, demand charge of \$160/kw/yr, energy charge of 15 mills/kwh and capacitor cost of \$7/kvar. The feeder data file is in Tables 1. The results of Capsize simulation are in Tables 2 and 3 and figure 3. The concentrated load results refer to the case where the zone loads are concentrated at their feed points. The main feeder is then analyzed as one radial feeder with no zones. This case gives insight to into the performance of the system when the zone concept is dismissed.

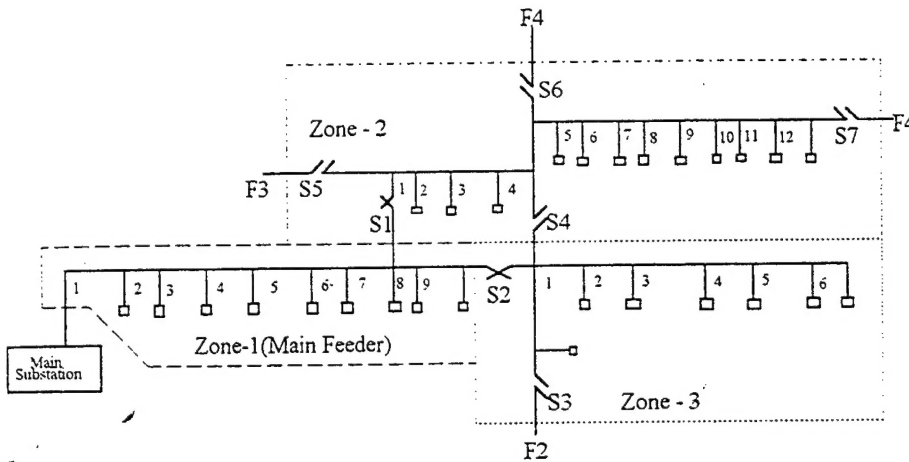


Figure 2: One Line Diagram of Test Feeder

Table 1 - Feeder Data File

SN	CONDUCTOR	R	X	L	P	Q	COMMENTS
1	477MCM/ACSR	.216	.4298	.1477	39	29	
2	477MCM/ACSR	.216	.4298	.2424	20	15	
3	477MCM/ACSR	.216	.4298	.1742	13	15	
4	477MCM/ACSR	.216	.4298	.0796	13	10	
5	477MCM/ACSR	.216	.4298	.1091	82	62	
6	477MCM/ACSR	.216	.4298	.1326	53	40	
7	477MCM/ACSR	.216	.4298	.1061	82	62	
8	477MCM/ACSR	.216	.4298	.0379	275	206	
9	477MCM/ACSR	.216	.4298	.0379	137	103	
10	477MCM/ACSR	.216	.4298	.0758	417	313	
11	477MCM/ACSR	.216	.4298	.0379	39	29	
12	477MCM/ACSR	.216	.4298	.0379	11	8	
0	Z-2, Lat-0	2	7	0	0	2	end of Zone-2
1	477MCM/ACSR	.216	.4298	.2803	1276	957	
2	477MCM/ACSR	.216	.4298	.0455	49	37	
3	477MCM/ACSR	.216	.4298	.1440	33	25	
4	477MCM/ACSR	.216	.4298	.0834	116	87	
5	477MCM/ACSR	.216	.4298	.0834	33	25	
6	477MCM/ACSR	.216	.4298	.2197	67	50	
0	Z-3, Lat-0	3	9	0	0	3	end of Zone-3

1	477MCM/ACSR	.216	.4298	.4152	14	10	
2	477MCM/ACSR	.216	.4298	.0452	33	25	
3	477MCM/ACSR	.216	.4298	1.3258	16	12	
4	477MCM/ACSR	.216	.4298	.1288	124	93	
5	477MCM/ACSR	.216	.4298	.0610	76	57	
6	477MCM/ACSR	.216	.4298	.0985	49	37	
7	477MCM/ACSR	.216	.4298	.0910	88	66	
8	477MCM/ACSR	.216	.4298	.1061	281	211	
9	477MCM/ACSR	.216	.4298	.1136	391	293	
0	Zone-1(Main Feeder)	0	0	0	0	0	end of Zone-1

**Legend for Table 1:**

SN - Section Number

R - Resistance in Ohms/Unit Length

X - Reactance in Ohms/Unit Length

L - Section Length in Miles

P - Real Power of Load in KW

Q - Reactive Power of Load in KVAR

**Table 2 - Capacitor Size, Location, Loss Reduction and Savings(Option - 1)**

Zone Number	Distance from Source in Miles	Size in KVAR	Peak Power Loss in KW	Peak Power Loss Reduction in KW	Savings in \$/yr
Concentrated Load	2.3852	2861.7	83.4	31.87	8877.5
Zone - 2	3.3361	888.4	3.5	1.23	342.5
Zone - 3	2.6655	977.5	1.8	0.65	180.6
	2.7110	75.9			
	3.0218	154.4			
Zone - 1(Main Feeder)	2.2716	894.6	83.4	32.21	8976.7

Given in Table 2 is the theoretical capacitor sizes and locations with the greatest savings. Practically, capacitor banks of less than 150 KVAR's are not economically feasible to install. Therefore, some of the smaller banks will have to be lumped. Option - 2. is used for this purpose and the results of Option - 2 CapSize simulation are given in Table 3. Figure 3 depicts the improved voltage profile due to installing shunt capacitors.

**Table 3 - Capacitor Size, Location, Loss Reduction and Savings(Option - 2)**

Zone Number	Distance from Source in Miles	User Defined Size in KVAR	Peak Power Loss in KW	Peak Power Loss Reduction in KW	Savings in \$/yr
Concentrated Load	2.3852	2850	83.4	31.86	8876.7
Zone - 2	3.2982	900	3.5	1.22	341.8
Zone - 3	2.6655	1050	1.8	0.62	174.3
Zone - 1(Main Feeder)	2.2716	1050	83.4	32.19	8970.5

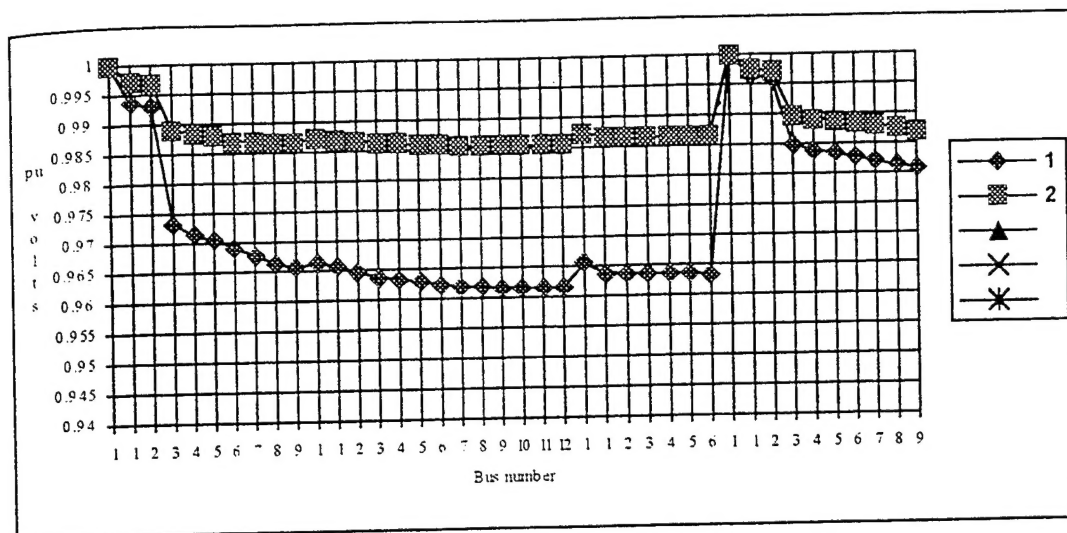


Fig 3: Voltage Profile of Feeder without and with capacitor  
 Graph 1: Original Feeder without capacitor. Graph 2 : Feeder with capacitor

Graph Identification:	Starting point	Section Starts at	Section Ends at	Comments
	Extreme Left	1	9	Concentrated Load
	After 9	1	12	Zone-2
	After 12	1	6	Zone - 3
	After 6	1	9	Zone-1(Main Feeder)

## 5. Conclusion

A rigorous design methodology for ohmic loss reduction in distribution feeders is presented. Feeders with laterals are considered through the zones method. A computer program called CapSize allows three design options. These options provide a powerful tool for tailoring the design in a progressive way to meet practical constraints.

## 6. Acknowledgement

This research was funded partially by the Strategic Environmental Research and Development Program (SERDP).

## 7. References

- [1] T.S. Abdel-Salam, et.al., "A new Technique For Loss Reduction using Compensation Capacitors Applied to Distribution Systems with varying Load Conditions", IEEE Summer Power Meeting, 1993.
- [2] M.M.A. Salama and A.Y. Chikhani, "A Simplified network approach to the var control problem for radial distribution systems", IEEE trans. on Power Delivery, vol.8, no.3, July 1993, pp.1529-1535.

- [3] J.J Grainger and S.H. Lee, "Capacity Release by Shunt Capacitor Placement On Distribution Feeders: A new Voltage-Dependent Model", IEEE trans. on Power Apparatus and Systems, vol.101, n0.5, May 1982, pp. 1236-1244.
- [4] J.J. Grainger et.al., "Optimal Design and Control Scheme for Continuous Capacitive Compensation of Distribution Feeders", IEEE trans. on Power Apparatus and Systems, vol.102, no.10, October 1983, pp. 3271-3278.
- [5] J.J. Grainger, et.al., "Optimal Capacitor Placement on Three-Phase Primary Feeders: Load And Feeder Unbalance Effect", IEEE trans. on Power Apparatus and Systems, vol.102, no.2, October 1983, pp. 3296-3305.
- [6] J.J. Grainger and S.H. Lee, "Optimal Sizing and Location of Shunt Capacitors for Reduction of Losses in Distribution Feeders", IEEE trans. on Power Apparatus and Systems, vol.100, March 1981, pp. 1105-1118.
- [7] Dariush Shirmohammdi and H.Wayne Hong, "Reconfiguration of Electric Distribution Networks for resistive Line Losses Reduction", IEEE trans. on Power Delivery, vol.4, no.2, April 1989, pp. 1492-1498.